LESSON 5: FIBONACCI NUMBERS

Collected and created by: Teacher Trần Hữu Hiếu

The Fibonacci Sequence is the series of numbers:

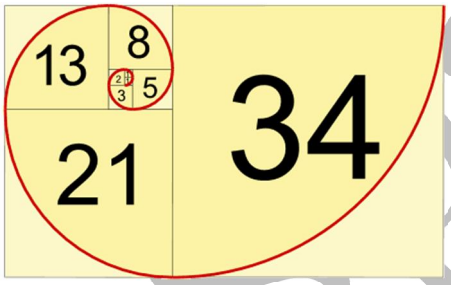
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The next number is found by adding up the two numbers before it.

+ The 2 is found by adding the two numbers before it (1+1)

+ Similarly, the 3 is found by adding the two numbers before it (1+2), And the 5 is (2+3), and so on!

When we make squares with those widths, we get a nice spiral:



Do you see how the squares fit neatly together?

For example 5 and 8 make 13, 8 and 13 make 21, and so on.

The Fibonacci Sequence can be written as a "Rule":

n =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
x_n =	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	...

Example: the **8th** term is the **7th** term plus the **6th** term:

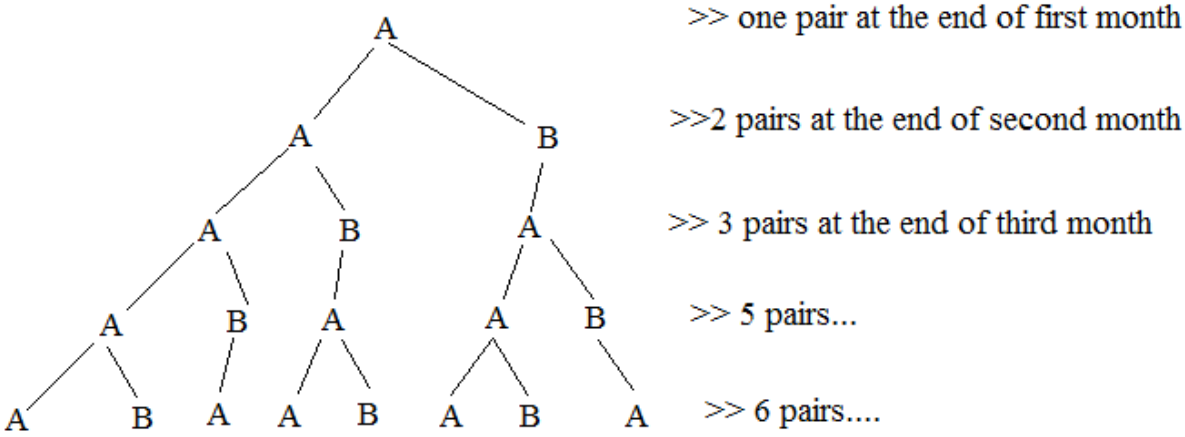
$x_8 = x_7 + x_6$

x_5	x_6	x_7	x_8	x_9	x_{10}
5	8	13	21	34	55

$x_8 = x_7 + x_6$

Example 1: A pair of rabbits can reproduce a pair of baby rabbits every month. The baby rabbits take a month to mature before they can reproduce another pair of baby rabbits. How many pairs of rabbits will there be by the end of one year?

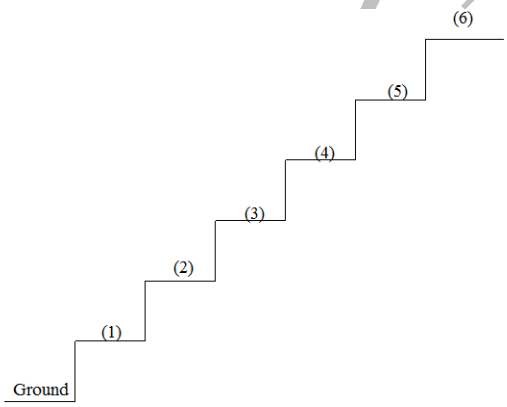
Solution: Let A denote a pair of adult rabbits and B denote a pair of baby rabbits.



So we have: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

There will be 233 pairs of rabbits by the end of one year!!!

Example 2: There is a flight of 6 steps in a staircase. Dave can take single or double steps each time when he climbs the staircase. In how many ways can he complete the climbing?



Solution:

To reach step (1): 1 way (1 single-step)

To reach step (2): 2 ways (2 single-steps or 1 double-step)

To reach step (3): 3 ways (3 single-steps / 1 double-step followed by 1 single-step/ 1 single-step followed by 1 double-step)

To reach step (4): 5 ways

Consider the following rules: In order to reach step (4), the last time Dave can climbs is in two different ways:

- reach step (2) and followed by a double-step.

- reach step (3) and followed by a single-step.

Number of way to reach step (4) = Number of way to reach step (3) + Number of way to reach step (2).

We have shown that this is a set of Fibonacci numbers.

1, 2, 3, 5, 8, 13, 21, ...

=> Dave can complete the climbing in 13 ways.

Problem 1: If the first three Fibonacci numbers are given as 1, 1 and 2, then what is the eleventh number?

Problem 2: If the first three Fibonacci numbers are given as $x_1 = 1$, $x_2 = 1$ and $x_3 = 2$, then what is the least value of n for which $x_n > 500$?

Problem 3: The partial sums of the first n and n + 1 numbers of the Fibonacci sequence are both divisible by 11. What is the smallest value of n for which this is true?

Problem 4: Is it possible for a Fibonacci number greater than 2 to be exactly twice as big as the Fibonacci number immediately preceding it? Explain why or why not.

Problem 5: Find the missing numbers in each of the following:

a) 3, 6, ( ), ( ), 24, ( ), ...

b) 3, 5, ( ), ( ), 21, ( ), ...

Problem 6: A sequence follows the pattern of Fibonacci numbers. Find the value of a if the value of the 6^{th} term equals 50 in: 5, a, 5 + a, ...

Problem 7: Mary plans to save *a* amount of money in January, and increase to *b* amount of money in February. The amount of money she plans to save in March will be the sum of *a* and *b*. The money she saves in April will be the amount of money she saves in February and March, and so on. Find the value of *a* and *b* if Mary saves $75 in June. Assume *a* and *b* are whole numbers.

Problem 8: Find the value of a in the following sequence. The value of sixth term equals 60.

a, 3, a + 3, a + 6, ...

Problem 9: Find the value of b in: 3, b, 3 + b, 3 + 2b, ...if the sixth term is 84.

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Problem 10: There are 12 matchsticks on a table. In how many ways can you draw the matchsticks if you can draw one or two matchsticks each time?

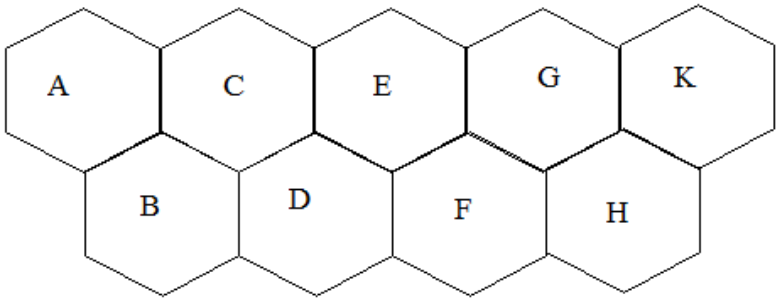
Problem 11: How many sequences of 1's and 2's sum to 14?

Problem 12: There is a flight of 8 steps in a staircase. A person can either take one or two steps up each time. In how many ways can the person reach the eighth step?

Problem 13: Peter bought a pair of baby rabbits in January. The baby rabbits grew up to adult rabbits in the second month and became productive in the third month. How many pairs of rabbits would there be in 12 months?

Problem 14: The second term of a Fibonacci number is 12. Its eight term is 180. Find the value of its first term?

Problem 15: In how many ways can a bee reach cell K if it start its journey from either cell A or cell B?



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FIBONACCI NUMBERS

VOCABULARY

No	Words	Meaning
1	numbers	số
2	In how many ways ...	Có bao nhiêu cách ...
3	Cell	ô
4	Rabbits	Con thỏ
5	Amount	Một lượng...
6	Either ... or	Hoặc ... hoặc ....
7	Pair	Cặp...
8	Follow / followed	Theo sau / theo sau bởi...
9	Step	bước
10	Staircase	Cầu thang
11	Reach	Đạt đến... tiếp cận đến, đạt được ..
12	(Sixth) term	Số hạng thứ (6)
13	Immediately Preceding	Ngay trước
14	Preceding	Trước
15	possible	Có thể

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