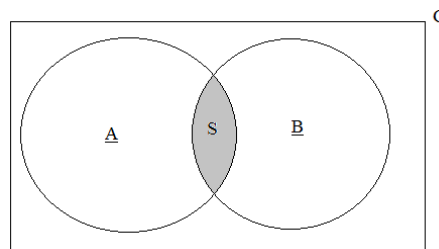


LESSON 4: VENN DIAGRAMS

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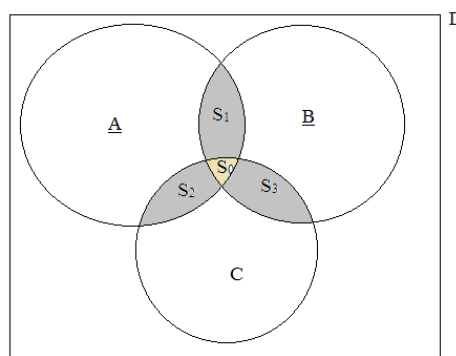
A **Venn Diagram** can be used to organize counting problems where some items are included in multiple groups and others are excluded.

Examples of a Venn diagram are shown below. The Venn diagram below shows the relationship between two different items A and B. The shaded area, S, is a region that is common to both A and B.



Total number of items or events, $C = A + B - S$

The Venn diagram below shows the relationship between three items or events.



The shaded area, S_1 , is common to both A and B.

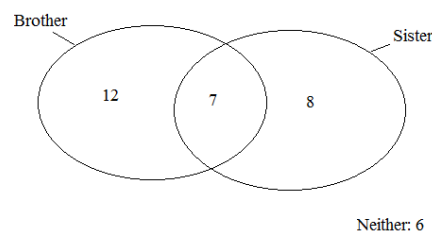
The shaded area, S_2 , is common to both A and C.

The shaded area, S_3 , is common to both B and C.

The shaded area, S_0 , is common to A, B and C.

Total number of items or events, $D = A + B + C - S_1 - S_2 - S_3 + S_0$

Problem 1. In science classroom: 19 students have a brother, 15 students have a sister, 7 students have both a brother and a sister, and 6 student don't have any sibling at all. How many students are in the classroom?

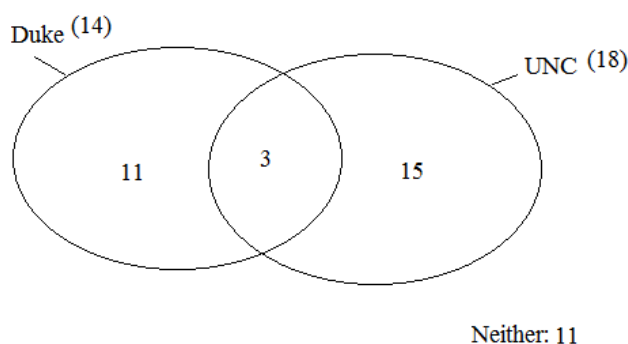


Reasoning: When you fill-in a Venn diagram to solve a problem, work from the inside out.

First, we fill-in a 7 where the two region overlap. Then we can fill in 12 students who only have a brother (19 minus the 7 we already accounted for) and 8 students who only have a sister. Six student have no sibling for a total: $7 + 12 + 8 + 6 = 33$ students.

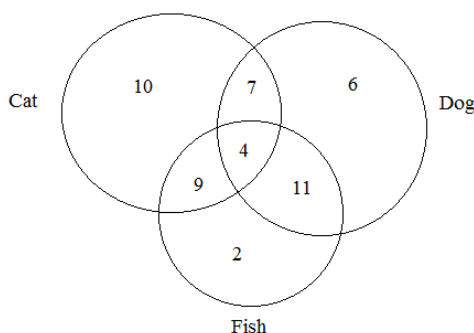
Problem 2. On a survey of 40 Raleigh students, 14 students responded that they like Duke, 18 students responded that they like UNC, and 11 students do not like Duke or UNC. How many students like both Duke and UNC?

Reasoning: This problem can be approached with a diagram, but it can also be solved logically. Drawing the diagram will help you understand the logic: there are 40 students, but $14 + 18 + 11 = 43$, so there must be three student who got counted twice. These are the three students in the overlapping portion of the Venn diagram who like both Duke and UNC.



Problem 3. Every student who applied for admission to a veterinary school has at least one pet: 30 have a cat, 28 have a dog and 26 have fish. If 13 students have fish and a cat, 15 students have fish and a dog, 11 students have a cat, a dog and fish. How many students applied to veterinary school?

Reasoning: Begin at the center of the diagram below and work your way out to get:



Using a Venn diagram is easy enough (there are 49 students), but you can also use some basic reasoning.

30 have cats + 28 have a dog + 26 have fish = 84 owners. Students who own two (or more) different animals were counted multiple times, so we subtract them: $84 - 13 - 15 - 11 = 45$. This seem right, but it is different from the answer we got with the diagram.

Notice that when we subtracted students with two pets, we subtracted the students who have all three pets three times. (We actually wanted to subtract them twice). Add these 4 student back to get 49.

Practice

Problem 4. At the pound there are 40 dogs. If 22 dogs have spots and 30 dogs have short hair, what is the fewest number of dogs that can have short hair and spots?

Problem 5. In a class of 50 students, 18 take Chorus, 26 take Band, and 2 take both Chorus and Band. How many students in the class are not enrolled in either Chorus or Band?

Problem 6. In a school of 320 students, 85 students are in the band, 200 students are on sports teams, and 60 students participate in both activities. How many students are involved in either band or sports?

Problem 7 A veterinarian surveys 26 of his patrons. He discovers that 14 have dogs, 10 have cats, and 5 have fish. Four have dogs and cats, 3 have dogs and fish, and one has a cat and fish. If no one has all three kinds of pets, how many patrons have none of these pets?

Problem 8 A guidance counselor is planning schedules for 30 students. Sixteen students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. Five say they want to take both French and Latin, and of these, 3 wanted to take Spanish as well. Five want only Latin, and 8 want only Spanish. How many students want French only?

Problem 9 In a class there are:

- 8 students who play football and hockey
- 7 students who do not play football or hockey
- 13 students who play hockey
- 19 students who play football

How many students are there in the class?

Problem 10 In a class there are 30 students.

- 21 students like Maths
- 16 students like English
- 6 students don't like Maths or English

How many students like both Maths and English?

Problem 11 In a family of six, everybody plays football or hockey.

4 members of the family play both sports and 1 member of the family plays only hockey.

Problem 12 John's mum buys 5 portions of chips. All the portions have salt or vinegar on them. Some have salt and vinegar. There are 2 portions with salt and vinegar and one portion with only vinegar. How many portions have only salt on them?

Problem 13 In a class of 32 pupils, 20 say that they like pancakes and 14 say that they like maple syrup. There are 6 pupils who do not like either.

How many of them like both pancakes and maple syrup?

Problem 14 There are 20 people in a room.

Of these, 15 are holding newspapers and 8 are wearing glasses.

Everyone wears glasses or holds a newspaper.

Problem 15 A pencil case contains 20 pens that are red or blue.

Of these, 8 are blue and 8 do not work. How many of the blue pens do not work if there are 6 red pens that do work?

Problem 16 In a school canteen there are 45 children.

There are 16 who have finished eating.

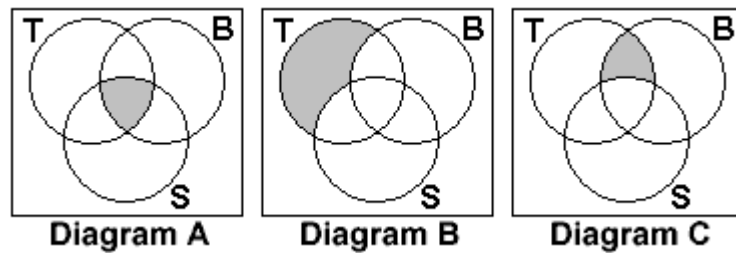
The others are eating either fish or chips, or both fish and chips.

There are 26 eating chips and 17 eating fish.

- How many are eating fish and chips?
- How many are eating fish without chips?
- How many are eating only chips?

Problem 17 The diagrams below represent the activities chosen by youth club members.

They can choose to play tennis (T), badminton (B) or squash (S)



Decide which diagram has the shading which represents:

- those who play all three sports
- those who play tennis and badminton, but not squash
- those who play only tennis

Problem 18 All the members of a group of 30 teenagers belong to at least one club.

There are 3 clubs, chess, drama and art.

- 6 of the teenagers belong to only the art club.
- 5 of the teenagers belong to all 3 clubs.
- 2 of the teenagers belong to the chess and art clubs but not to the drama club.
- 15 of the teenagers belong to the art club.
- 2 of the teenagers belong only to the chess club.
- 3 of the teenagers belong only to the drama club.

- How many of the group do chess and drama but not art?
- How many of the group belong to the chess club?

Problem 19. Ten friends go out to dinner together: 7 order an appetizer, 5 order a soup, and 4 order a salad. If everyone orders something but no one orders exactly two things, how many people order all three things?

Problem 20. How many of the smallest 1,000 positive integers are divisible by 5, 6 or 7?

VENN DIAGRAM VOCABULARY

No	Words	Meaning
1	Diagram	Biểu đồ
2	relationship	mối liên hệ
3	different items	đôi tượng/mục khác nhau
4	events	sự kiện
5	shaded area	vùng bôi xám
6	account	đếm/ tính
7	multiple	gấp lên một số lần (bội số.../nhân thêm..)
8	subtract	trừ
9	add	cộng
10	belong to	thuộc về
11	represents	đại diện / mô tả / thể hiện
12	participate	tham gia
13	activity	hoạt động
14	approach	tiếp cận
15	exactly	chính xác