Manhattan Mathematical Olympiad 2000
Grades 5-6

Put your name on all papers you use and turn them all in.

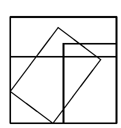
Try to solve as many problems as you can, in any order you choose. For any problem you try, give as complete an answer as you can. Include a clearly written explanation of how you found your answer and why it is true. You may use drawings or calculations as part of your justification.

Problem 1. Jane and John wish to buy a candy. However Jane needs seven more cents to buy the candy, while John needs one more cent. They decide to buy only one candy together, but discover that they do not have enough money. How much does the candy cost?

Problem 2. Farmer Jim has an 8 gallon bucket full with water. He has three empty buckets: 3 gallons, 5 gallons and 8 gallons. How can he get two volumes of water, 4 gallons each, using only the four buckets?

Problem 3. A pizza is divided into six slices. Each slice contains one olive. One plays the following game. At each move it is allowed to move an olive on a neighboring slice. Is it possible to bring all the olives on one slice by exactly 20 moves?

Problem 4. Three rectangles, each of area 6 square inches, are placed inside a 4 in. by 4 in. square. Prove that, no matter how the three rectangles are shaped and arranged (for example, like in the picture below), one can find two of them which have a common area of at least 2/3 square inches.



1. Write each of your solutions on a separate piece of paper.

2. Write your name, address and the name of your school at the top of each piece of paper you turn in.

3. Explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2001

Grades 5-6

1. Piglet added together three consecutive whole numbers, then the next three numbers, and multiplied one sum by the other. Could the product be equal to 111,111,111?

2. The dates of three Sundays of a month were even numbers. What day of the week was the 20th of the month?

3. Is it possible to divide 5 apples of the same size equally between six children so that no apple will be cut into more than 3 pieces? (You are allowed to cut an apple into any number of equal pieces).

4. You have a four-liter jug and a six-liter pot (both of cylindrical shape), and a big barrel of water. Can you measure exactly one liter of water?

1. Write each of your solutions on a separate piece of paper.

2. Write your name, address and the name of your school and school teacher at the top of each piece of paper you turn in.

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MANHATTAN MATHEMATICAL OLYMPIAD 2002

Grades 5-6

1. You are given a rectangular sheet of paper and scissors. Can you cut it into a number of pieces all having the same size and shape of a polygon with five sides? What about polygon with seven sides?

2. One out of every seven mathematicians is a philosopher, and one out of every nine philosophers is a mathematician. Are there more philosophers or mathematicians?

3. Let us consider all rectangles with sides of length a, b both of which are whole numbers. Do more of these rectangles have perimeter 2000 or perimeter 2002?

4. Somebody placed digits $1, 2, 3, ..., 9$ around the circumference of a circle in an arbitrary order. Reading clockwise three consequitive digits you get a 3-digit whole number. There are nine such 3-digit numbers altogether. Find their sum.

Manhattan Mathematical Olympiad 2003
Grades 5-6

Write each solution on a separate piece of paper. Write your name, address and the name of your school and school teacher at the top of each paper you turn in. Explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

Problem 1. Cut the triangle shown in the picture into three pieces and rearrange them into a rectangle.

Problem 2. Prove that no matter what digits are placed in the four empty boxes, the eight-digit number

$$\mathbf{9999}\square\square\square\square$$

is not a perfect square. (A *perfect square* is a whole number times itself. For example, 25 is a perfect square because $25 = 5 \times 5$.)

Problem 3. Two players play the following game, using a round table 4 feet in diameter, and a large pile of quarters. Each player can put in his turn one quarter on the table, but the one who cannot put a quarter (because there is no free space on the table) loses the game. Is there a winning strategy for the first or for the second player?

Problem 4. Form an eight-digit number, using only the digits $1, 2, 3, 4$, each twice, so that: there is one digit between the 1's, there are two digits between the 2's, there are three digits between the 3's, and there are four digits between the 4's.

The 8^{th} Manhattan Mathematical Olympiad

April 3, 2004

Grades 5-6

Put your name, address, name of your school, and name of your teacher on all papers you use, and turn them all in.

Below are **four** problems. Try to solve as many problems as you can, in any order you choose. For any problem you try, give as complete an answer as you can. Include a clearly written explanation of how you found your answer and why it is true. You may use drawings or calculations as part of your justification.

1. Is there a whole number, so that if we multiply its digits we get 528?

2. Can you form six squares with nine matches? How about fourteen squares with eight matches? (It is assumed that all matches have equal length, and you cannot break any of them.)

3. There are 169 lamps, each equipped with an on/off switch. You have a remote control that allows you to change exactly 19 switches at once. (Every time you use this remote control, you can choose which 19 switches are to be changed.)

(a) Given that at the beginning some lamps are on, can you turn all the lamps off, using the remote control?

(b) Given that at the beginning all lamps are on, how many times do you need to use the remote control to turn all lamps off?

4. An elevator in a 100 floor buidling has only two buttons. The UP button makes the elevator go 13 floors up, and the DOWN button makes the elevator go 8 floors down. Is it possible to go from 13^{th} floor to 8^{th} floor?

1. Write your solutions on separate pieces of paper.

2. Write your name, your address, name of your school, and the school teacher at the top of each piece of paper you turn in.

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MANHATTAN MATHEMATICAL OLYMPIAD 2005

Grades 5-6

1. Is there a whole number which becomes exactly 57 times less than itself when one crosses out its first digit?

2. What is the largest number of Sundays can be in one year? Explain your answer.

3. An alien from the planet Math came to Earth on Monday and said: A. On Tuesday he said AY , on Wednesday AY Y A, on Thursday AY Y AY AAY . What will he say on Saturday?

4. The parlament of the country Ar consists of two houses, upper and lower, both have the same number of people. The law says that each member must vote ”Yes” or “No”. One day, when all members of both houses were present and voted on an important issue, the speaker informed the press that the number of members voted ”Yes” was greater by 23 than the number of members voted ”No”. Prove that he made a mistake.

1. Write your solutions on separate pieces of paper.

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MANHATTAN MATHEMATICAL OLYMPIAD 2006

Grades 5-6

1. Is it possible to place six points in the plane and connect them by non-intersecting segments so that each point will be connected with exactly

a) Three other points?

b) Four other points?

2. Martian bank notes can have denomination of $1, 3, 5, 25$ marts. Is it possible to change a note of 25 marts to exactly 10 notes of smaller denomination?

3. What is bigger:

$99 \cdot 99 \cdot .... \cdot 99$ (product of 20 factors) or

$9999 \cdot 9999 \cdot ... \cdot 9999$ (product of 10 factors)?

4. How many whole numbers, from 1 to 2006, are divisible neither by 5 nor by 7?

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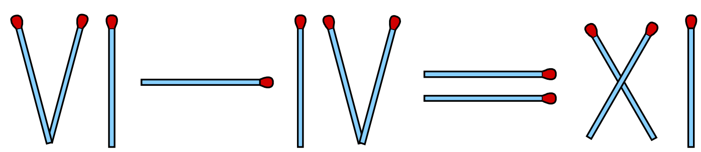
MANHATTAN MATHEMATICAL OLYMPIAD 2007

Grades 5-6

1. There are 64 cities in the country Moonland. Prove that there will be at least three of them which will have the same number of rainy days in September 2007.

2. Matches from a box are placed on the table in such a way that they form a (wrong) equality in Roman numbers (each segment on the picture below is a single match). Change a position of exactly one match (without removing or breaking it) and get a correct equality in Roman numbers:



3. A craftsman has 4 oz of paint in order to paint all faces of a cube with the edge equal to 1 in. He cuts the cube into 27 smaller identical cubes. How much more paint does he need in order to paint completely faces of all smaller cubes?

4. Arrange the whole numbers 1 through 15 in a row so that the sum of any two adjacent numbers is a perfect square. In how many ways this can be done?

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MANHATTAN MATHEMATICAL OLYMPIAD 2008

Grades 5-6

1. What is the minimal number of acute triangles one can cut the regular polygon with 2008 edges? Justify your answer.

2. A boy has as many sisters as brothers, but each sister has only half as many sisters as brothers. How many brothers and sisters are in the family?

3. The teacher asked each of four children to think of a four-digit number. "Now please transfer the first digit to the end and add the new number to the old one. Tell me your results".

Mary: 8,612

Jack: 4,322

Kate: 9,867

John: 13,859.

"Everyone except Kate is wrong", said the teacher. How did he know?

4. There are three closed boxes on a table. It is known that one contains two black balls, another contains one black and one white ball, and the third one contains two white balls. Each box has a sticker: "Two whites", "Two blacks", "One white and one black". It is known that all stickers are wrong. How can one place stickers on the boxes correctly by taking just one ball from one box, and not looking inside?

1. Write your solutions on separate pieces of paper.

2. Write your name, your address, name of your school, and the school teacher at the top of each piece of paper you turn in.

3. When solving a problem explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2009

Grades 5-6

1. Insert the pluses between (some of) the digits of 987654321 to get the total of 99.

2. 30 students from five grades donated 40 toys. It is known that students from the same grade donated equal number of toys, while students from different grades donated different number of toys. How many students donated exactly one toy?

3. A nonstop train leaves New-York for Boston at 60 miles per hour. Another nonstop train leaves Boston for New-York at 40 miles per hour. How far apart are the trains 1 hour before they pass each other? You may assume that the railroad is a straight segment.

4. A square carpet of the size 4×4 meters contains 15 holes (you may assume that the holes are dots). Prove that one can cut out from it a carpet of the size 1×1 meter which does not contain holes inside.

Manhattan Mathematical Olympiad 2010
Grades 5-6

1. Number 23 is written on the blackboard. Every minute the following procedure is performed: the written number is erased and the product of its digits plus 12 is written on its place (for instance, after first minute the number $2 \times 3 + 12 = 18$ will be written). What number will be on the blackboard after one hour?

2. A student took a test consisting of 20 problems. Each correct solution gives him 8 points, for each incorrect solution he gets *negative* 5 points. For a problem which he did not try to solve he receives 0 points. The student got the total of 13 points. How many problems did he try to solve?

3. Is it possible to place 25 pennies on a table such that each of them touches exactly three others?

4. Prove that among all people on earth there are two which have the same number of friends (Note: A is a friend of B if B is a friend of A).

Solutions 2010

5-6.1. The sequence starts as 23, 18, 20, 12, 14, 16, 18, 20, . . . and then it repeats every 5 minutes. Thus after 60 minutes the number written will be the same as after 5 minutes, which is 16.

5-6.2. He tried 13. Let A be the number of correct solutions and B - of the incorrect ones. Then we must have $A + B \leq 20$ and $8A - 5B = 13$. In the equation we consider remainders after dividing by 5. 8A has the same remainder as 13, that is 3. In the sequence of values 8A for A = 0, 1, 2, . . . only

$8 \cdot 1 = 8, 8 \cdot 6 = 48, 8 \cdot 11 = 88, \ldots$

have the remainder 3. Solving $8A - 5B = 13$ for B gives the possible (A, B) pairs:

$(1, -1),(6, 7),(11, 15), \ldots$

The larger A's would give larger B's. But already 11+15 > 20. On the other hand

$B = -1$ cannot happen as well. Thus the only solution is A = 6, B = 7.

Hence the student tried 6 + 7 = 13 problems.

5-6.3. No. Let's count the number of touching points. Every coin has 3, but also every touching point belong to exactly two coins. So altogether we must have $1/2 \cdot 3 \cdot 25$ touches, which is not an integer number, hence impossible.

Remark: Geometry and size of coins is irrelevant here.

5-6.4. Let the total number of people on earth be N. Each can have between 0 and $N - 1$ friends. Suppose no two have the same number of friends. Then every number 0, 1, 2, . . . , $N - 1$ occurs exactly once as a number of someone's friends. Hence, there is someone, say Joe, who has no friends. On the other hand there is someone who is a friend with everybody, in particular with Joe.

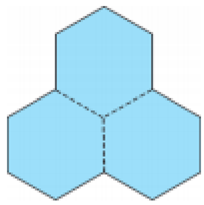
Contradiction.

Manhattan Mathematical Olympiad

April 16, 2011

Grades 5-6

1. Can you cut the figure on the right into three congruent pieces and then put them back together to form one regular hexagon?



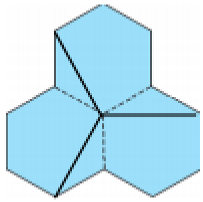
2. John and Mary play in a very long chess tournament. John plays every 16th day, while Mary plays every 25th day. Will they sometime have to play in two consecutive days?

3. Prove that at least one of any 18 successive three-digit numbers is divisible by the sum of its digits.

4. 30 children from school went to a museum in pairs. After visiting the museum they went back to school also in pairs (possibly different). Show that upon their arrival back to class it is always possible to divide them into three groups such that in any group no two kids were a pair on either way.

Solutions 2011

5-6.1. Make three cuts from the center to the second farthest vertices of little hexagons in a consistent manner (see the picture). The side length of the big hexagon is √3 times the side length of the little hexagons.



5-6.2. Yes, they will have to, because 16 and 25 are relatively prime numbers. So no matter how far apart they started to play, there will be a day on which they both play. And then 175 days later Mary will play and 176 days later John will play.

5-6.3. Among any 18 successive numbers there are at least two which are divisible by 9, and, moreover, one of them is even, that is divisible, in fact, by 18. The sum of its digits is either 9 or 18 and hence divides the number.

5-6.4. We will divide the kids into 2 groups such that no two in the same group were a pair. Then the problem with 3 groups (in fact with any number of groups) is solved by further splitting any of the groups in two. Consider the graph of relations: nodes are students and two students are connected with an edge if they were a pair on either way. Each student is connected to at most two others. Thus the graph decomposes into a collection of simple cycles each of even length (because in each cycles students were split in pairs, say on a way to the museum). In each chain we put children into the groups by alternate order.

MANHATTAN MATHEMATICAL OLYMPIAD 2012

Write the solutions on separate paper. Start each problem with new page. You can do problems in any order you like. NO calculators allowed. You have to justify all your answers with clear arguments. Solution will be available after 2 pm at

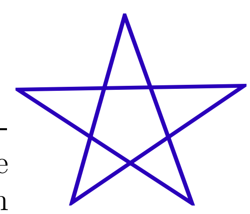
www.math.ksu.edu/events/hscomp/olympiad

Grades 5-6

1. How many five-digit numbers are there which will produce the number 2012 if one digit is crossed out?

2. A classroom floor is colored using two colors. Prove that there are two identically colored points exactly 1 foot apart.

3. Can you draw a closed
a) 6-segment
b) 7-segment
c) 8-segment
polygonal line which self-crosses each of its segments exactly once? Example on the picture shows a 5-segment closed polygonal line which self-crosses each of its segments *twice*.



4. Jack wants to enter a wonder cave. In front of the entrance there is a round table with 4 identical hats lying symmetrically along the circle. There are 4 identical coins, one under each hat. Jack can lift any two hats, examine the two coins, turn them as he likes and put the hats back. After this Jack closes his eyes and the table starts spinning, and when it stops Jack cannot tell by how much the table rotated. Then again he can choose two hats and so on. The wonder cave will open if and only if the coins are either all HEADS or all TAILS up. How must Jack act to enter the cave?

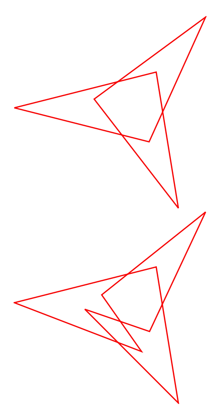
Remark: Lifting hats randomly and turning all coins, say, HEADS up is not a winning strategy. Jack may be so unlucky that he never lifts a hat which covers TAIL.

Solutions

5-6.1. $9+4\cdot10-4\cdot1=45$. Here 9 is for choice of the first digit, and each of the 10's for the choice of placing a new digit in the other 4 possible position. However, say adding the digit 2 in front or in between first and second digits will result in the same 5-digit number. And same for any other digit of 2012. Thus we must subtract 4.

5-6.2. Place an equilateral triangle with side length 1 foot somewhere in the room. Then at least 2 of its vertices will be of same color. They are exactly 1 foot apart.

5-6.3. a) and c). Yes. See the picture.
b) No. Each self-crossing point belongs to two segments, so the number of segments has to be even.



5-6.4. Here is the strategy. First Jack chooses two hats opposite each other and turn coins HEADS up. Next move he choose two neighboring hats and turns the coins HEADS up again. If the cave does not open, it means that three of the hats have HEADS and one has TAIL. Next he choose two opposite hats. If one of the coins is TAIL then he flips it and wins. Otherwise he turn one of the coins to TAIL. Now we have two neighbor HEADS and the other two neighbors - TAILS.

MANHATTAN MATHEMATICAL OLYMPIAD 2013

Write the solutions on separate paper. Start each problem with new page. You can do problems in any order you like. NO calculators allowed. You have to justify all your answers with clear arguments. Solution will be available next week at

www.math.ksu.edu/events/hscomp/olympiad

Grades 5-6

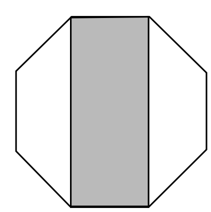
1. There is a 12 liter jar full with water. There are also two empty jars, 5 and 8 liters. Can you divide the water into

a) 3 and 9 liters?

b) 6 and 6 liters?

2. Is it possible to represents the number 2013 as a sum of (at least two) positive integer numbers such that their product is also equal to 2013?

3. Two diagonals are are drawn in a regular octagon (see picture). Prove that the area of the shaded rectangle equals the sum of the areas of the remaining trapezoids.



4. Tom and Jerry play the following game. Tom has some number of coins and Jerry has none. Jerry can take any (non-zero) number of coins from Tom. Then Tom can take some (again, non-zero) number of coins back, but necessarily a different number. Then again Jerry takes some from Tom, but necessarily a number which did not occur before. And so on. The game stops when someone cannot make a move. What is the largest number of coins Jerry can have at the end if

a) Tom had 13 coins at the beginning?

b) Tom had 50 coins at the beginning?

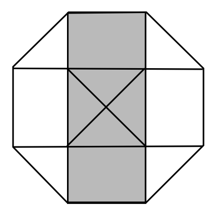
Solutions 2013

5-6.1. (a) Yes. Pour 8 liters into the 8-liter jar. Then pour 5 liters from 8-liter jar into the 5-liter jar. And pour those 5 liters back into the 12-liter jar. We have 9 liters in the 12-liter jar and 3 liters in the 8-liter jar.

(b) Yes. Continue from (a). Pour 3 liters from the 8-liter jar into the 5-liter jar. Then pour 8 liters from the 12-liter jar to fill up the 8-liter jar (1 liter is left in the 12-liter jar). Then pour 2 liters from the 8-liter jar to fill up the 5-liter jar. And finally pour those 5 liters back into the 12-liter jar.

5-6.2. Yes. For example, $2013 = 671 \cdot 3 \cdot 1 \cdot \dots \cdot 1$ and $2013 = 671 + 3 + 1 + \dots + 1$, there are 1339 1's in both expressions.

5-6.3. See the picture. The two pairs of rectangles are equal, and the four unshaded triangles are equal to the four shade triangles in the central square.



5-6.4. (a) 13. The point is to be not greedy. Jerry takes 2 coins, then Tom has to take 1 back. Then Jerry takes 3, Tom has to take all 4 back. Then Jerry takes 6, Tom has to take 5 back. Then Jerry takes 7, Tom has to take all 8 back. And one more cycle like this. After 12 moves, all numbers between 1 and 12 has been used. Finally Jerry takes all 13 coins.

(b) 49. Jerry follows the above strategy. Let's prove that Jerry can't have all 50 coins. Suppose he could, then his move would be the the last. But then there would be odd number of moves in total. Hence, one of the numbers between 1 and 50 did not occur. Tom could take that number of coins back. Contradiction.

MANHATTAN MATHEMATICAL OLYMPIAD 2014

Write the solutions on separate paper. Start each problem with new page. You can do problems in any order you like. NO calculators allowed. You have to justify all your answers with clear arguments. Solution will be available next week at

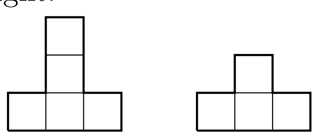
www.math.ksu.edu/events/hscomp/olympiad

<u>Grades 5-6</u>

1. A dandelion blooms in two stages: first it becomes yellow for three days, and then it turns white for two more days and then it stops blooming. On Monday there were 20 yellow and 14 white dandelions on Jeremy's front yard. On Wednesday there are 15 yellow and 11 white. How many white dandelions will be on Jeremy's front yard on the following Saturday?

2. There are several kids in a room. If Ann gives half of her candies to Victor, then everyone will have the same amount of candies. If Ann gives all her candies to John, then John will have as many candies as all other kids combined. How many kids are there in the room?

3. Draw a figure which can both be cut into 4 figures of the shape on the left and in 5 figures of the shape on the right.



4. Ann and Peter are playing the following game. There are two numbers written on the blackboard in the beginning: 2014 and 2015. The players take turns and on each step one the following operations can be performed:

(1) The player can subtract one of the digits written on the blackboard from one of the numbers.

(2) The player can divide one of the numbers by two, given that it is even.

The first player to get a number less than 10 wins. Ann goes first. Which player has a winning strategy?

MANHATTAN MATHEMATICAL OLYMPIAD 2014
Solutions

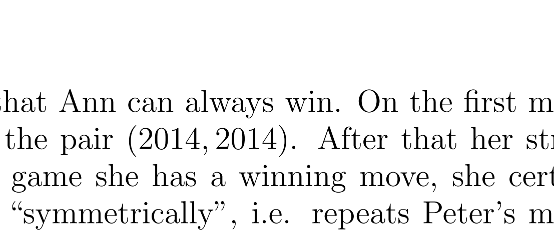
5-6.1. 11 dandelions which are white on Wednesday turned white either on Wednesday or on Tuesday. Therefore, they were yellow on Monday. The remaining $20 - 11 = 9$ dandelions which were yellow on Monday are still blooming on Wednesday as well, so they have to remain yellow. Consecutively, they would turn white on Thursday and stop blooming by Saturday.

There are $15-9 = 6$ more yellow dandelions on Wednesday. They started blooming after Monday, and, therefore, would still be blooming on Saturday, but all of them turn white by that time. We thus accounted for all dandelions which started blooming on Wednesday or earlier. But those that start blooming later could not turn white by Saturday.

Answer: there will be 6 white dandelions on Saturday.

5-6.2. Ann gives half of her candles to Victor and all become equal. This means that Victor had no candles and Ann had twice as many candles as anybody else. In particular, all other kids had the same number of candles. Now by receiving all Ann's candles John will have 3 times more candles than anybody else (except Ann and Victor). So there are 3 more kids in the room. Thus there are 6 kids altogether.

5-6.3. Here is one possible picture



5-6.4. We will show that Ann can always win. On the first move Ann can substruct 1 from 2015 and get the pair $(2014, 2014)$. After that her strategy is as follows. If at some point in the game she has a winning move, she certainly does that move. Otherwise, she plays "symmetrically", i.e. repeats Peter's moves, so that after her move the numbers are equal. Note that she can always do that, because all the digits that were on the blackboard when Peter made his move are still on the blackboard after his move (remember that before Peter's move the numbers are equal). Note also that with this strategy Peter will never get a winning move. Indeed, if Peter got

MANHATTAN MATHEMATICAL OLYMPIAD 2015

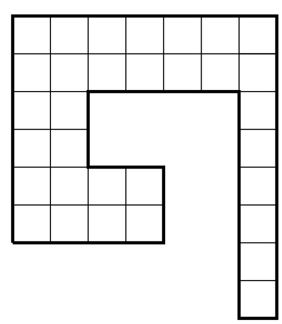
Write the solutions on separate paper. Start each problem with new page. You can do problems in any order you like. NO calculators allowed. You have to justify all your answers with clear arguments.

Grades 5-6

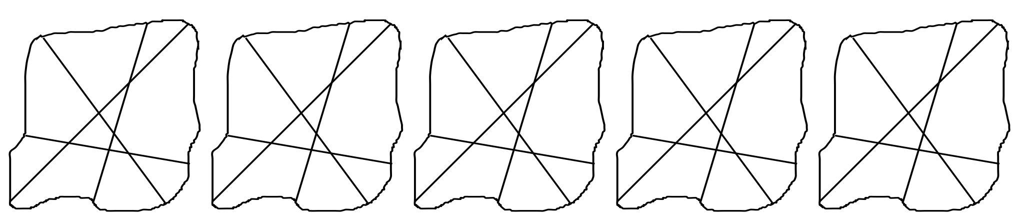
1. Is it possible to represent the number 203 as a sum of several whole numbers such that their product is also equal to 203?

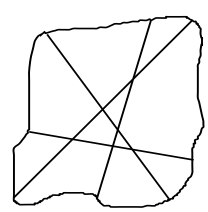
2. A monkey is happy if it eats 3 types of fruit. There are 20 pears, 30 bananas, 40 peaches and 50 oranges. How do you distribute the fruit to make the most number of monkeys happy? What is this number?

3. Cut the figure below in two identical figures.



4. There are 4 intersecting straight line paths in a garden, as shown in the pictures below. Place 4 trees so that there are equal number of trees (two) on both sides of EVERY path! (Practice and put your answer on the LAST picture.)





MANHATTAN MATHEMATICAL OLYMPIAD 2015
SOLUTIONS

Grades 5-6

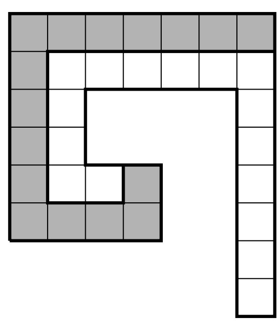
1. YES: The numbers we can use are 29, 7, $\underbrace{1, 1, \ldots, 1}_{167\ 1\text{'s}}$.

2. The answer is 45. First of all, there are $20 + 30 + 40 + 50 = 140$ fruits altogether, so if each monkey gets 3 fruits, there could be no more than 46 monkeys, because $3 \times 46 = 138$, but $3 \times 47 = 141 > 140$.

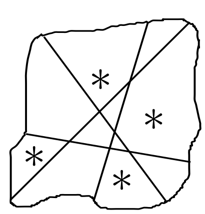
However, it is impossible to feed 46 monkeys, because that would require at most 46 oranges, which would diminish the total number of fruits used by at least 4 (oranges), so at most 136 fruit would be used.

How can we feed exactly 45 monkeys? Feed 5 monkeys with pear-banana-orange; feed 15 monkeys with pear-peach-orange; feed 25 monkeys with banana-peach-orange.

3.



4.



MANHATTAN MATHEMATICAL OLYMPIAD 2016

Write the solutions on separate paper. Start each problem with a new page. You can do problems in any order you like. NO calculators allowed. You have to justify all your answers with clear arguments. Solution will be available at

www.math.ksu.edu/events/hscomp/olympiad

Grades 5-6

1. Three excavators can dig 3 holes in 2 hours. How many holes can six excavators dig in 5 hours?

2. There are 5 apples, 8 pears, and 11 oranges in a basket. Every day Alice eats two different fruits. Can it happen that some day there are the same number of apples, pears and oranges remaining in the basket?

3. (a) Cover (without overlaps) the 8×8 chessboard by the triminos 3×1 such that there is one square left uncovered.
(b) Which square can it possibly be?

4. A family: Father, Mother, Grandfather and Daughter want to cross the Dark bridge. They have only 2 flashlights, so at most two people can cross at the same time.

- The father can cross in 1 minute.
- The mother can cross in 2 minutes.
- The daughter can cross in 5 minutes.
- The grandfather can cross in 10 minutes.

If two people are crossing, they go with the speed of the slowest of the two. Can the entire family cross in
(a) 19 minutes?
(b) 17 minutes?

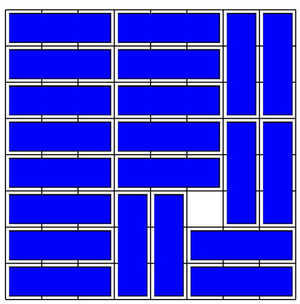
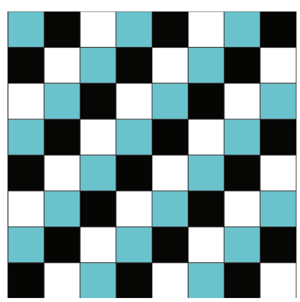
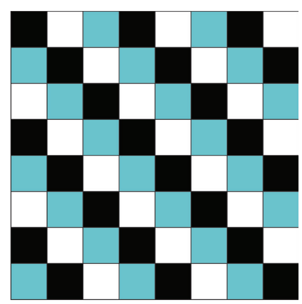
MANHATTAN MATHEMATICAL OLYMPIAD 2016
Solutions

5-6.1. Six excavators can dig 6 holes in 2 hours, that is, they dig 3 holes in 1 hour. So 6 excavators can dig 15 holes in 5 hours.

5-6.2. Yes. Here is a possible sequence
(5, 8, 11), (5, 7, 10), (5, 6, 9), (5, 5, 8), (4, 5, 7), (3, 5, 6), (3, 4, 5), (3, 3, 4), (2, 3, 3), (2, 2, 2).

5-6.3. (a) See picture.

(b) You can have only the squares c3, c6, f3 and f6 left uncovered. Here is the argument. Color the board into 3 colors (see picture). Then each trimino covers three different colors. Since there 21 (white), 21 (grey) and 22 (black), the remaining square has to be black. Now rotate the coloring by 90 degrees. The uncovered square has to be black again. The 4 squares c3, c6, f3 and f6 are the only squares which are black in both pictures.

5-6.4. (a) (Easy) Father takes everyone, by one at a time.

(b) (Harder) Here is a possible sequence of crossings:

- Father and Mother cross, Father comes back (2+1 minutes).
- Grandfather and Daughter cross, Mother comes back (10+2 minutes).
- Father and Mother cross (2 minutes).

MANHATTAN MATHEMATICAL OLYMPIAD 2017

Write the solutions on separate pieces of paper. Start each solution with a new page. You can do the problems in any order you like. NO calculators allowed. You have to justify all your answers with clear arguments. Solution will be available at

www.math.ksu.edu/events/hscomp/olympiad

Grades 5-6

1. Three drawers, one with spoons, another with forks and the third one with a mixture of spoons and forks, are labeled "SPOONS", "FORKS" and "SPOONS AND FORKS". Its is known that all labels are incorrect. Can you tell what each drawer contains by just picking one item from one of the drawers (of your choice)?

2. Four distinct digits are written on two two-sided cards, one digit on each side of each card. Is it possible that all possible combinations of two-digit numbers that can be shown with these cards are prime? (Prime is a number whose only divisors are 1 and the number itself).

3. A $2 \times 2 \times 2$ cube is assembled of 8 cubes. The sides of smaller cubes are painted blue or red. One-third of all sides are painted blue and two-third are red. In the assembled cube two-third of the visible sides are blue and one-third are red. Show that you can reassemble the $2 \times 2 \times 2$ cube so that all visible sides are red.

4. The number 60 is written on a blackboard. Alice and Bob take turns subtracting from the number on the blackboard any of its divisors (including 1 or the number itself), and replacing the original number with the result of this subtraction. The player who writes the number 0 loses. Alice starts. Show that she can always win no matter what Bob's moves are.

MANHATTAN MATHEMATICAL OLYMPIAD 2017
Solutions

5-6.1. Yes. You can pick an item from the “SPOONS AND FORKS” drawer. If it is a spoon, then “SPOONS AND FORKS” contains spoons, “FORK” contains spoons and forks, and ”SPOONS“ contains forks. If it’s a fork, then “SPOONS AND FORKS” contains forks, “SPOONS” contains spoons and forks, and ”FORKS“ contains spoons.

5-6.2. This is impossible.

If one of the digits is even, then there is an even number two-digit number, which cannot be prime. Similar, there cannot be 5 among the digits. Thus, the digits have to be 1, 3, 7 and 9. 1 and 9 cannot be on two different cards, because 91 is not prime. On the other hand, 3 and 9 cannot be on different cards, because 39 is not prime. But 9 cannot be on the same card with 1 and 3 at the same time.

5-6.3. There are $8 \times 6 = 48$ sides altogether. 16 sides are blue and 32 are red. The assembled cube has $6 \times 4 = 24$ little sides of which two-thirds, that is 16, are blue. Thus all invisible sides are red. Then you can rotate each little cube to change all its invisible sides to become visible. Then in the resulting assembled cube all visible sides are red.

5-6.4. Alice always subtracts 1. Then Bob is left with an odd number and he has to subtract an odd divisor. Hence Alice always gets an even number, from which she again subtracts 1. Eventually Bob will end up with 0.