SHORT ANSWER PROBLEMS

Country:____________ Name: ____________________ ID:_____________ Score:_____

Instructions:

- Write down your name and country on the answer sheet.
- Write your answer on the answer sheet.
- For problems involving more than one answer, points are given only when ALL answers are correct.
- Each question is worth 1 point. There is no penalty for a wrong answer.
- You have 60 minutes to work on this test.
- Use black or blue colour pen or pencil to write your answer.

"Smart, Skilled, and Creative In a Joyful Competition for Excellence"

International Mathematics and Science Olympiad 2015

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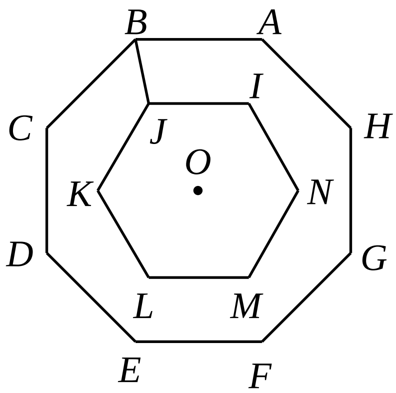
(1) Anne asks her teacher his age. He replied 'My age now is a square number but after my birthday it will be a prime number.' Assuming his age is below 65 and above 20, how old is he now?

(2) Given that $\overline{240a84} \times 234 = \overline{56b90256}$. What is the value of $a+b$?

(3) If I place all three operational symbols +, –, × in all possible ways into the blanks of the expressions 5____4____6____3, one symbol per one blank, each resulting expression will have a value. What is the largest of these values?

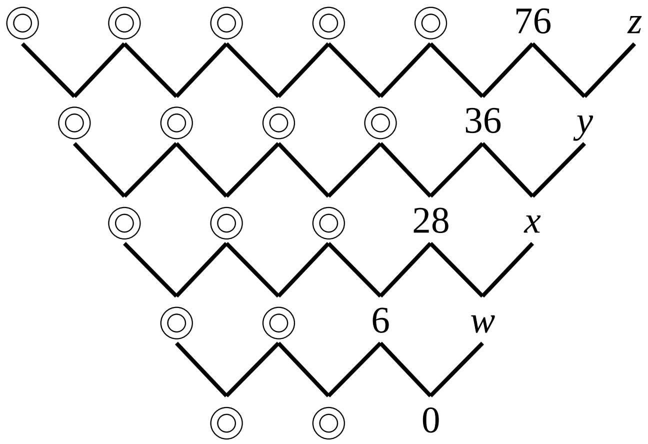
(4) In the diagram below, the regular octagon *ABCDEFGH* and the regular hexagon *IJKLMN* are centered around the same point *O* such that *AB* // *IJ*. If the measure of $\angle CBJ = 56°$, find the measure of $\angle BJK$, in degrees.



(5) A boy has a cup of tea and a girl has an empty glass having the same volume as the cup. In the first step, the boy pours $\frac{1}{2}$ of the tea from the cup into the glass. In the second step, the girl pours $\frac{1}{3}$ of the tea from the glass into the cup. In the third step, the boy pours $\frac{1}{4}$ of the tea from the cup into the glass. In the fourth step, the girl pours $\frac{1}{5}$ of the tea from the glass into the cup. This alternate pouring continues such that in each step, the denominator increases by 1. What fraction of the tea is in the cup after the thirteenth step?

(6) There is a committee of 5 members. The chairman will be seated in a permanent chair at the round table. In how many ways can the other 4 members be seated at the same table if there are exactly 8 chairs?

(7) In the arrangement below, each number is the non-negative difference of the two numbers above it. What is the average of the eight possible values of z?



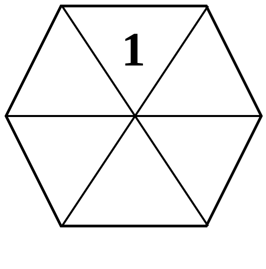
(8) How many positive integers from 1 up to 2015 are **not** divisible by any of the following numbers: 2, 20, 201 and 2015?

(9) In a survey of 100 students, 84 said they disliked playing Tennis, 74 said they disliked skiing, 62 students said they disliked both playing tennis and skiing. How many students liked both playing tennis and skiing?

(10) We know that 0, 2, 4, 6 and 8 are even digits. How many even digits are used from 1 to 100?

(11) Two numbers are called mirror numbers if one is obtained from the other by reversing the order of digits. For example, 123 and 321. If the product of a pair of mirror numbers is 146047, then what is the sum of this pair of mirror numbers?

(12) In the hexagon at the right, 1 is placed in the top triangle. In how many different ways can we place 2, 3, 4, 5 and 6 in the remaining empty triangles, such that the sum of the numbers in opposite triangles is 5, 7 or 9?



(13) Six identical pyramids with square bases are assembled to make a cube which has a volume 2744 cm^3 (Refer Figure 2). What is the length *OH*, which is the height from the vertex to the base square *ABCD* of each pyramid, in cm? (Refer Figure 1).

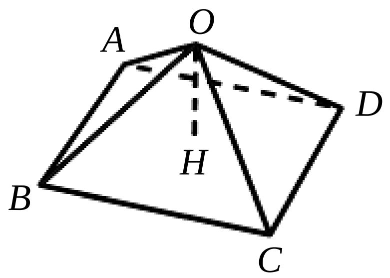


Figure 1

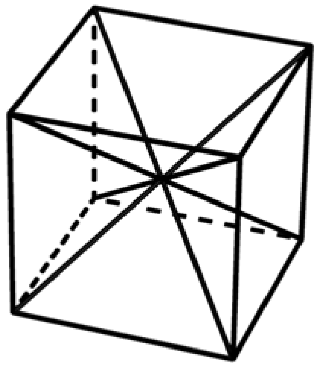


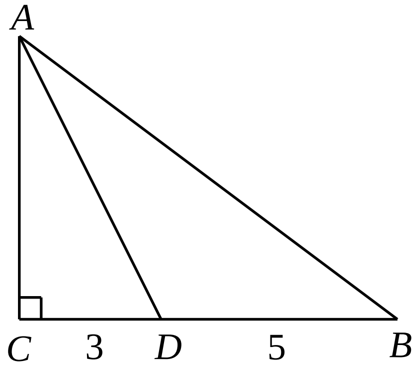
Figure 2

(14) How many of the following numbers 1×2, 2×3, 3×4, …, 29×30 are divisible by 3 or 5?

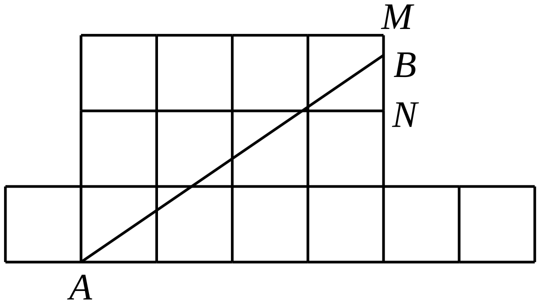
(15) A fox sees on its front a rabbit grazing 21 meters from his place. In one second, the rabbit runs 5 steps while the fox runs only 3 steps; it is also known that the distance travelled by the fox in 4 steps takes the rabbit 9 steps. If the distance the rabbit runs in every step is 0.6 meter, how many seconds will it take the fox to catch the rabbit?

(16) Consider all the 3-digit numbers such that its digits are all different and there is no digit “0” used. Find the sum of all such 3-digit numbers.

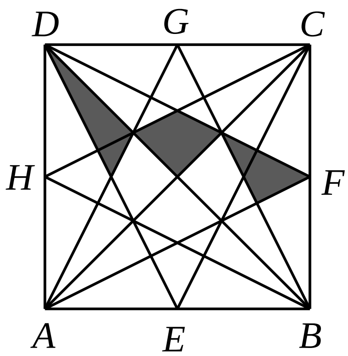
(17) *ABC* is a right triangle with $\angle C = 90°$. The bisector of $\angle A$ intersects *CB* at *D*. If $CD = 3\,\text{cm}$ and $BD = 5\,\text{cm}$, what is the length of *AB*, in cm?



(18) The diagram shown in the figure below composed of 15 unit squares. AB divides the area of the given figure into two equal parts. Find the value of $\frac{MB}{BN}$.

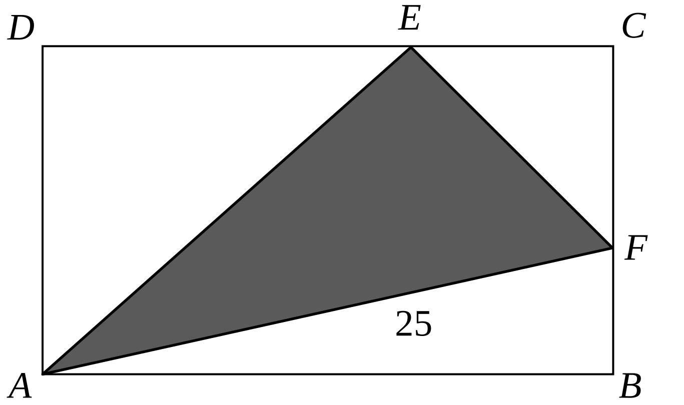


(19) The square $ABCD$ is drawn with points E, F, G and H as the midpoints of each side as shown in the figure below. If the total area of the shaded region is 15 cm^2, what is the area of the square $ABCD$, in cm^2?



(20) The symbols I, M, S, O, 1 and 5 are written in a row in some order.
(1) M is either the first or the last symbol from the left.
(2) S is the fourth symbol from the left.
(3) S is to the left, not necessarily immediately, of O.
(4) I is to the right, not necessarily immediately, of M.
(5) The symbols O and 1 are next to each other.
(6) There is exactly one other symbol between I and 1.
What is the symbol in the second place from the left?

(21) $ABCD$ is a rectangle with E as a point on CD and F is a point on BC such that $\angle AEF = 90°$ and $AF = 25\,\text{cm}$. The length of DE, EC, CF, FB, AE and EF are positive integers. What is the area of rectangle $ABCD$, in cm^2?



(22) There are some distinct positive integers whose average is 38 with 52 as one of those integers. If 52 is removed, the average of the remaining integers is 37. Find the largest possible positive integer in those integers.

(23) The sum of 47 distinct positive integers is 2015. At most how many of these positive integers are odd?

(24) ***Unit fractions*** are those fractions whose numerator is 1 and denominator is any positive integer. Express the number 1 as the sum of seven different unit fractions, given five of them are $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{9}$, $\frac{1}{15}$ and $\frac{1}{30}$. Find the product of the two remaining unit fractions.

(25) From the 99 positive integers less than 100, I chose as many different numbers as I could so that no subset of my numbers had a sum of 100. If the sum of all my numbers was as large as possible, what was the smallest number I actually chose?